

About My Mandelbrot Set and Julia Set Engine

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The Mandelbrot Set

The Mandelbrot set is the set of all points on the complex plane which stay bounded under iteration in the polynomial function $F(Z) \Rightarrow Z^n + C$; where “C” corresponds to the coordinates of the point being evaluated and “Z” is initialized to the origin (i.e. $0 + 0i$). The function is evaluated using these initial conditions and the output value is plugged back into “Z” with each additional iteration. As the function is iterated in this manner, the magnitude of its output value either stays bounded within a circle of radius 2 centered about the origin or it escapes the circle and “blows up” to infinity. When plotting the Mandelbrot set, usually a color is assigned to each point based on whether and how fast it escapes.

The Julia Sets

A Julia set is similar to the Mandelbrot set in that it is generated by iterating the function $F(Z) \Rightarrow Z^n + C$, but in the case of a Julia set, “C” is fixed at a particular value which is unique to that Julia set and “Z” is initialized to the coordinates of the point on the complex plane being evaluated. This means that there are an infinite number of different Julia sets (one for each unique value of “C”).

How the Two are Related

Because each unique Julia set is based on a specific value of “C”, the Mandelbrot set acts as a “map” of all of the Julia sets where the value of “C” for each Julia set corresponds to a specific location on the Mandelbrot set plot. There is a definite correlation between a location on the Mandelbrot set plot and the shape and structure of the Julia set associated with that value of “C”.

How this Excel Engine Works

This engine in excel uses columns and rows in the spreadsheet to represent the real and imaginary coordinates of evenly spaced points on the complex plane. It assigns a color to each cell based on how many iterations it takes for the output value of the function with an input of that complex value to reach a magnitude greater than 2. Cells that start out with a magnitude greater than 2 are colored dark blue. Cells whose value never exceed a magnitude of 2 are colored black. The rest of the cells are assigned a color on a blue, yellow, red color gradient scale based on a percentage of the total range of the values in the plot. This means that the color gradient rescales as you zoom into an area that has a densely-packed range of values. The large numbers of calculations are carried out in hidden rows within the spreadsheet and the resulting iteration count is reflected in a visible row at the top of each group of hidden rows. Colors are assigned using conditional formatting applied to both the background and the text in the cell.

The yellow cells in the upper left corner of the spreadsheet allow the user to select which type of plot to generate (Mandelbrot set or Julia set), an integer value for the order “n” of the polynomial function, coordinates for “C”, and a scale factor in order to “zoom in” or “zoom out”. A scale factor of 1 generates plots in which a circle of radius 2 fills the entire plot area. The maximum useful scale value is around 1E13 (10 trillion) at which point the resolution limit of the floating-point calculations in excel begins to produce “noise” in the image. The horizontal and vertical resolution is listed in the upper left corner and corresponds to the number of rows and columns used to generate the plot.

The coordinates of the graph center on the Mandelbrot set are always equal to the chosen value of “C” while the Julia sets are always plotted with the graph centered at the origin. The center of the graph is marked with a small white cross-hairs. The size of the cross-hairs roughly corresponds to 1/10 of the screen. This means that if you increase the scale by a factor of 10, the portion of the graph now covered by the cross-hairs will fill the entire screen.

How to Generate Plots

To generate a plot, select either “Mandelbrot” or “Julia”, type in the desired value for “n”, enter the desired real and imaginary values for “C”, input the desired scale factor, and then click “Calculate” in the lower left corner of the window. If the “Calculate” link doesn’t display in the lower left corner, you can either select it the “Function” -> “Calculate Sheet” drop-down menus at the top of the window or you can use the “FN” + “F9” keys. It may take a several minutes to complete the large number of calculations, but be patient, the results will be worth the wait! It is especially interesting to switch between the Mandelbrot set and Julia set for a chosen value of “C” to see how they relate to each other.

The main spreadsheet is locked to prevent inadvertent changes to the calculation cells, leaving only the input cells open for editing. There is no password set on the sheet protection so if you want to unlock the sheet to see how the calculations work, or to make your own modifications, simply unprotect the sheet.

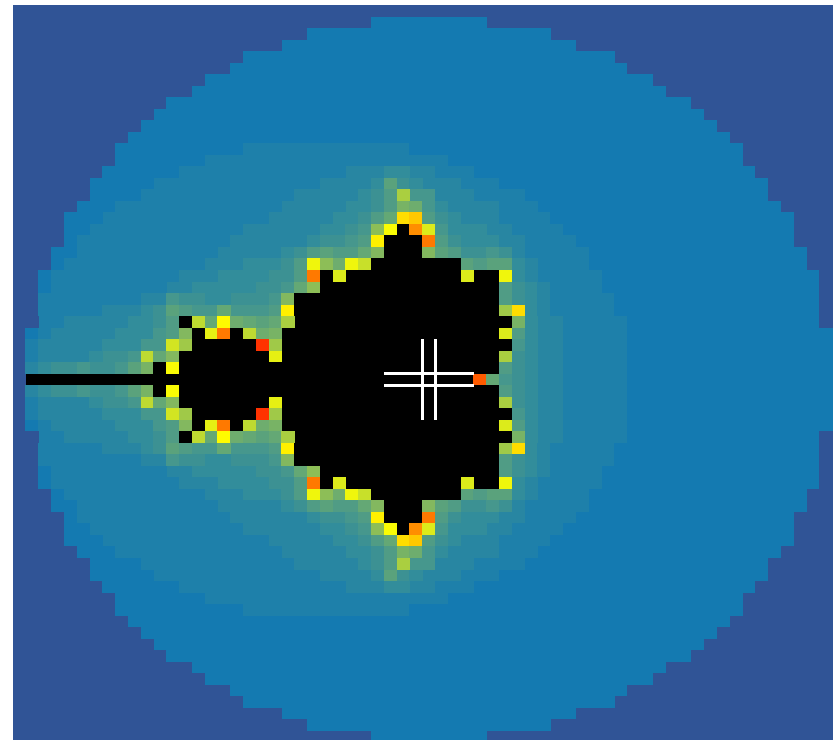
Extras

I have included an “Atlas” in a separate tab which contains a list of coordinates for “C” which I have found to be interesting and which generate plots with amazing detail at very high scale factors (depending on the iteration limit). To use these coordinates simply copy and paste them into the real and imaginary coordinate cells on the main spreadsheet.

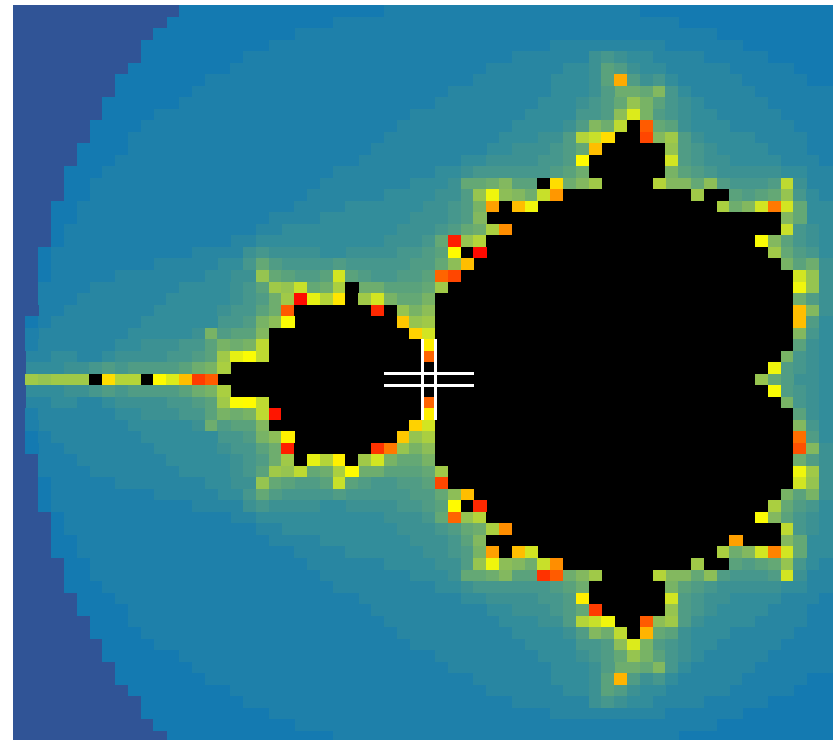
An interesting thing to note when plotting higher order polynomial functions is that the number of axes of symmetry in the Mandelbrot set is always one less than the order of its polynomial function (i.e. the cubic Mandelbrot set has double sided symmetry), whereas the number of axes of symmetry in the Julia sets match the order of its polynomial function (i.e. cubic Julia sets have three sided symmetry).

These spreadsheets are designed to allow them to be easily modified and scaled up or down in resolution by copying or deleting groups of rows or columns. If you change the resolution, the graph will re-scale to fit a space of that size.

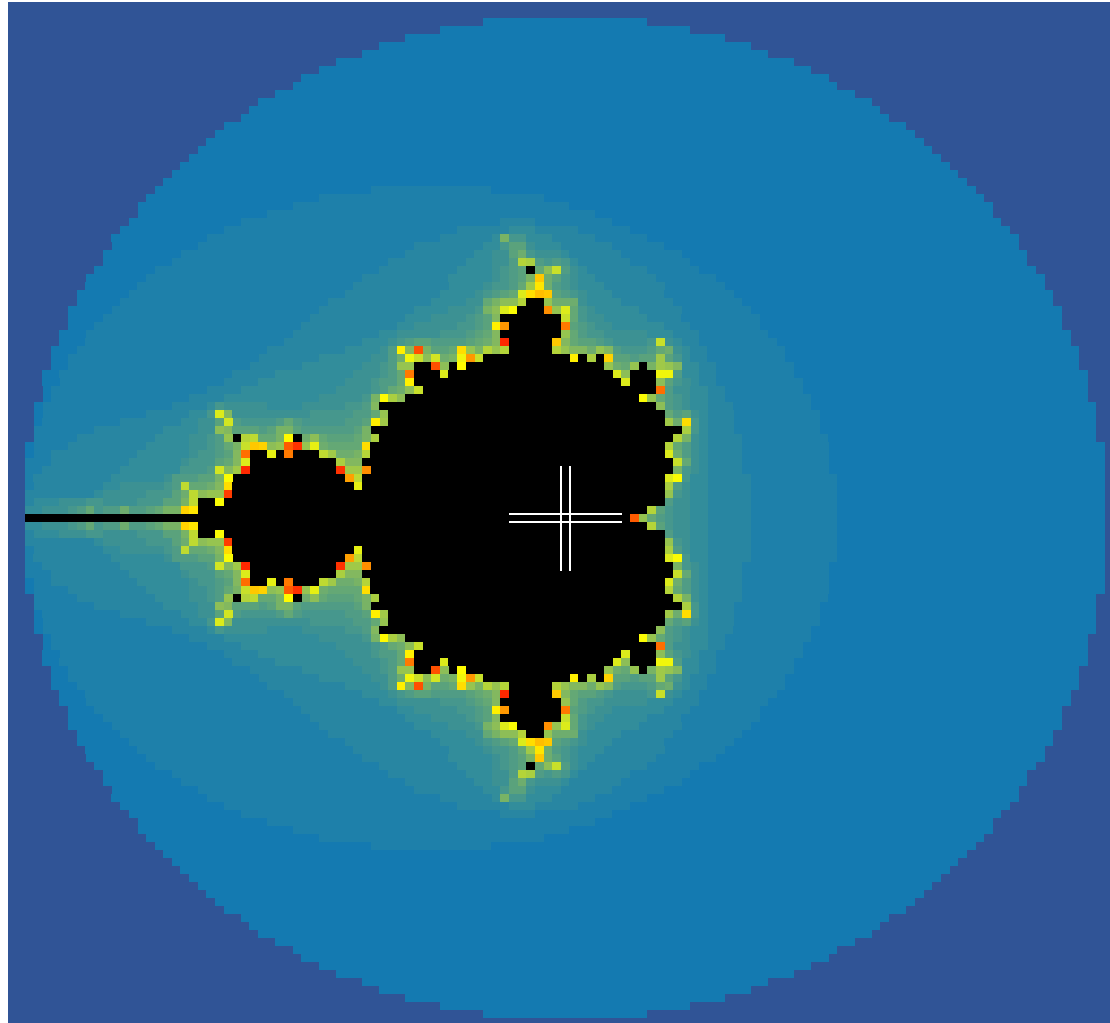
| | | |
|---------------------------------|--------------------|--------------------|
| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
| Resolution | 0.0000000000000000 | 0.0000000000000000 |
| 64 | Scale-> | 1.00E+00 |



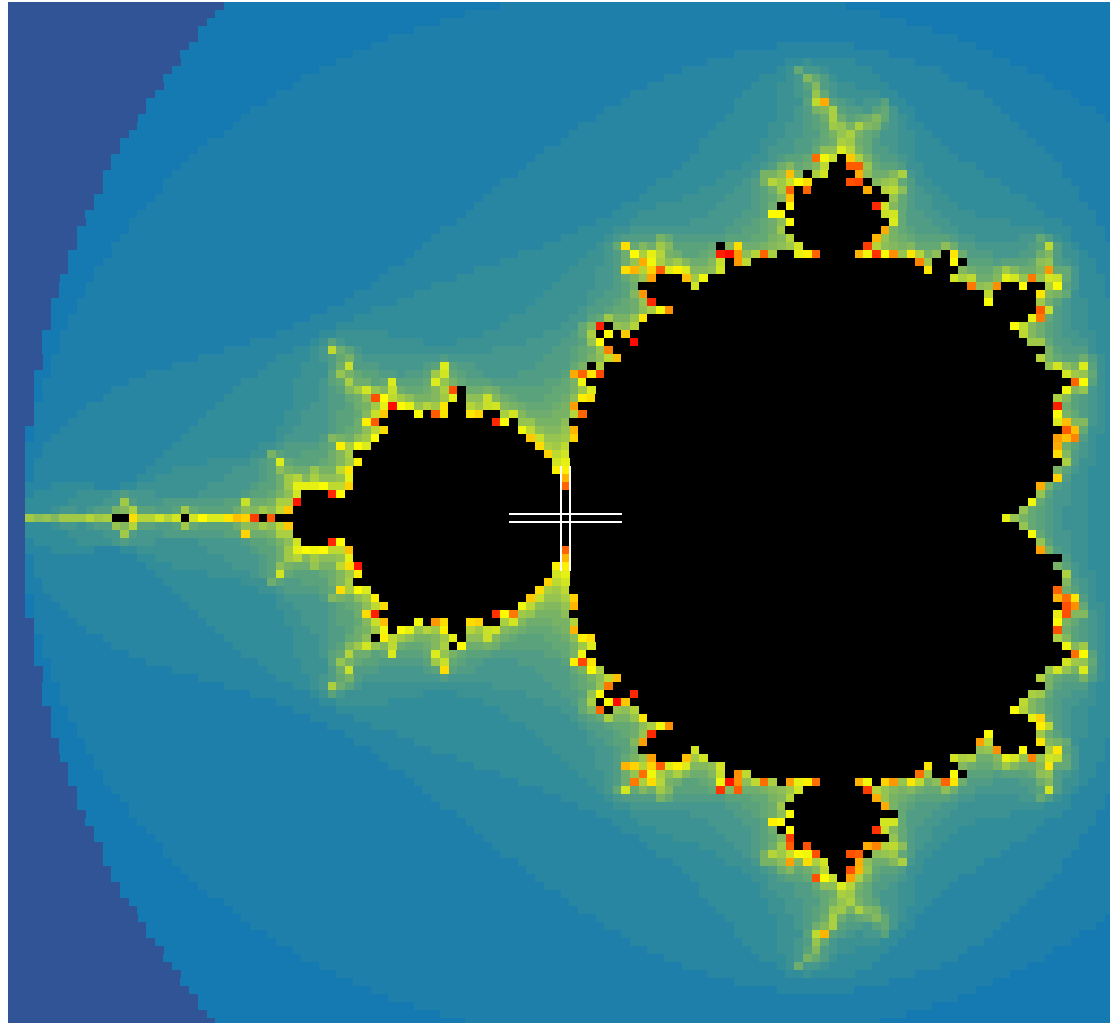
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| 64 | Scale-> | 1.60E+00 |



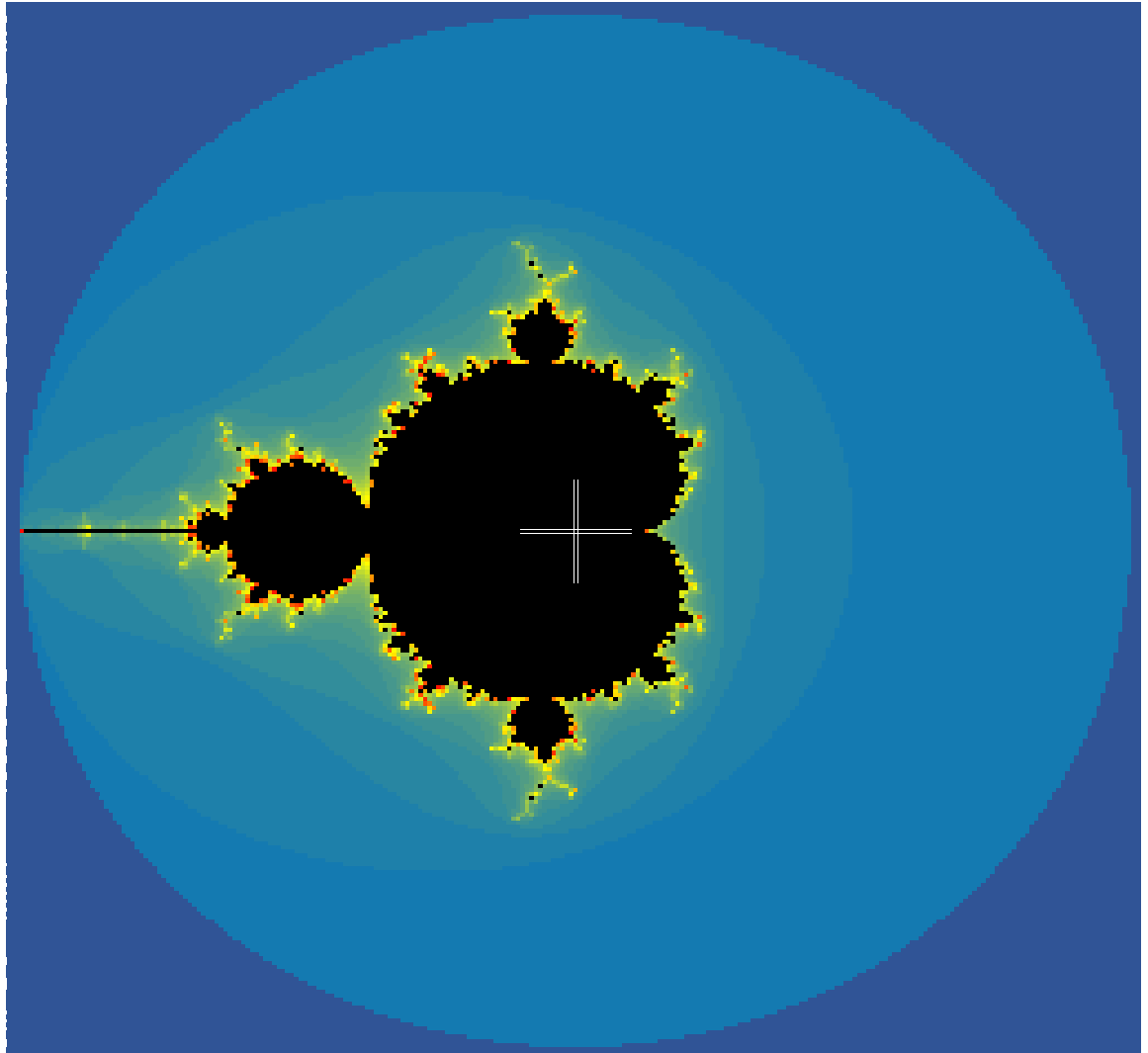
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|---------------------------------|--------------------|--------------------|
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| 128 | Scale-> | 1.00E+00 |



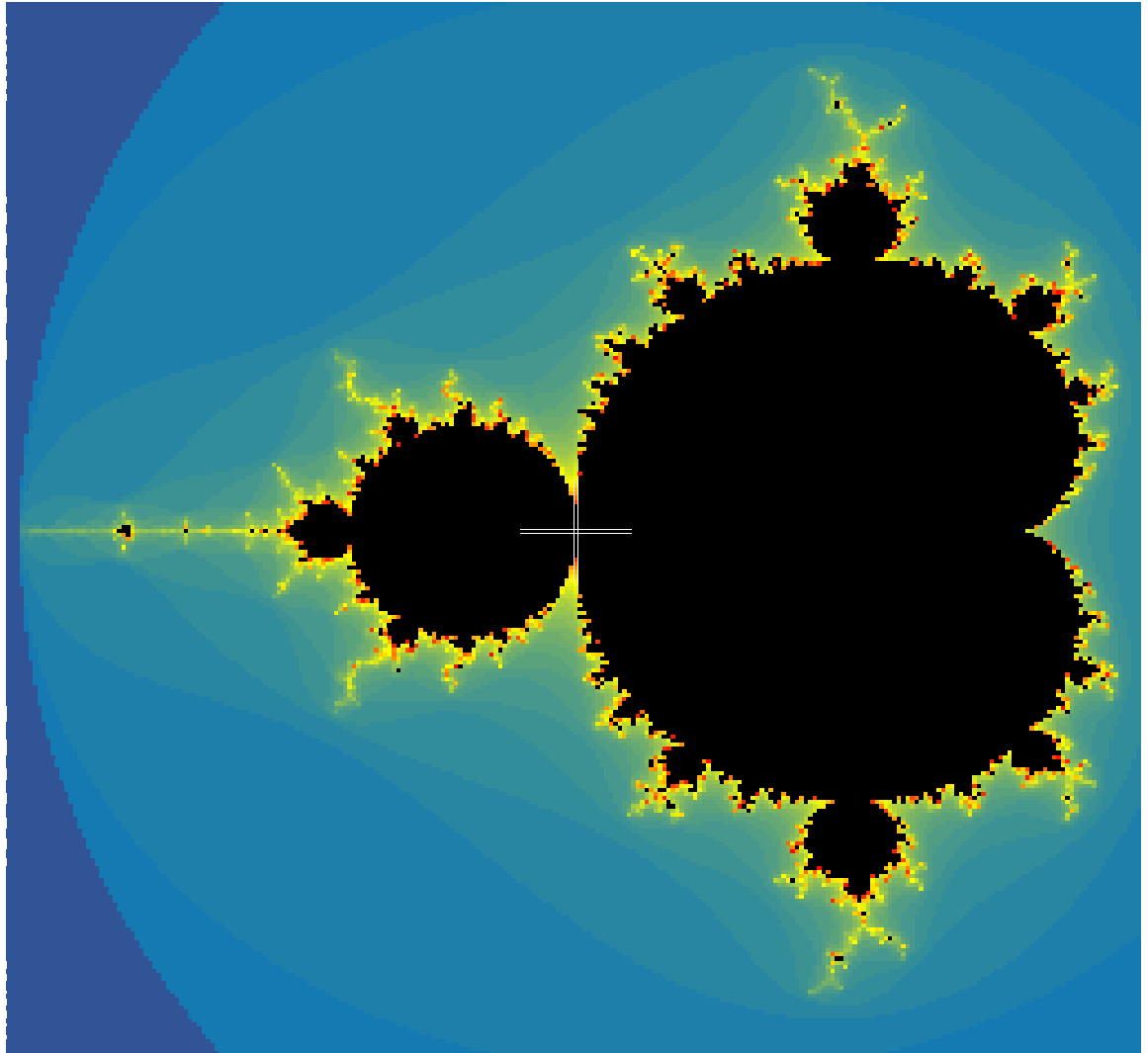
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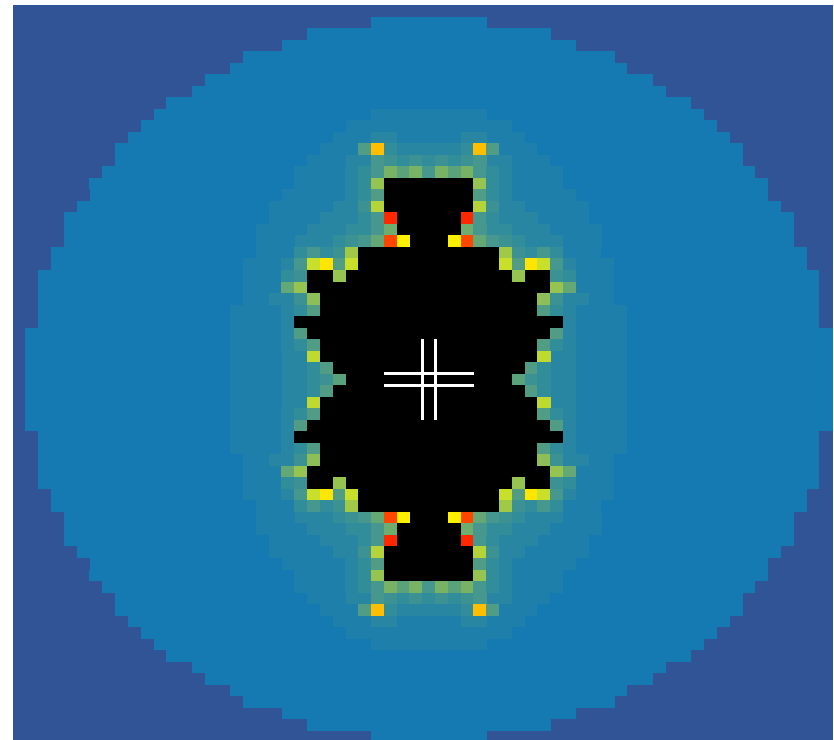
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| Polynomial Order (n) | C | |
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| Resolution | 0.000000000000000 | 0.000000000000000 |
| 256 | Scale-> | 1.00E+00 |



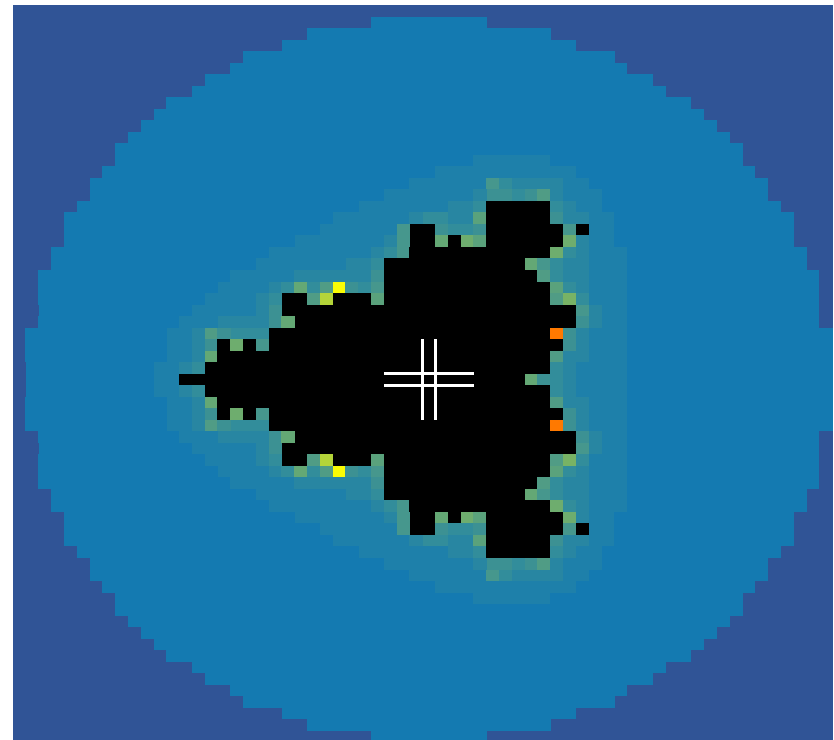
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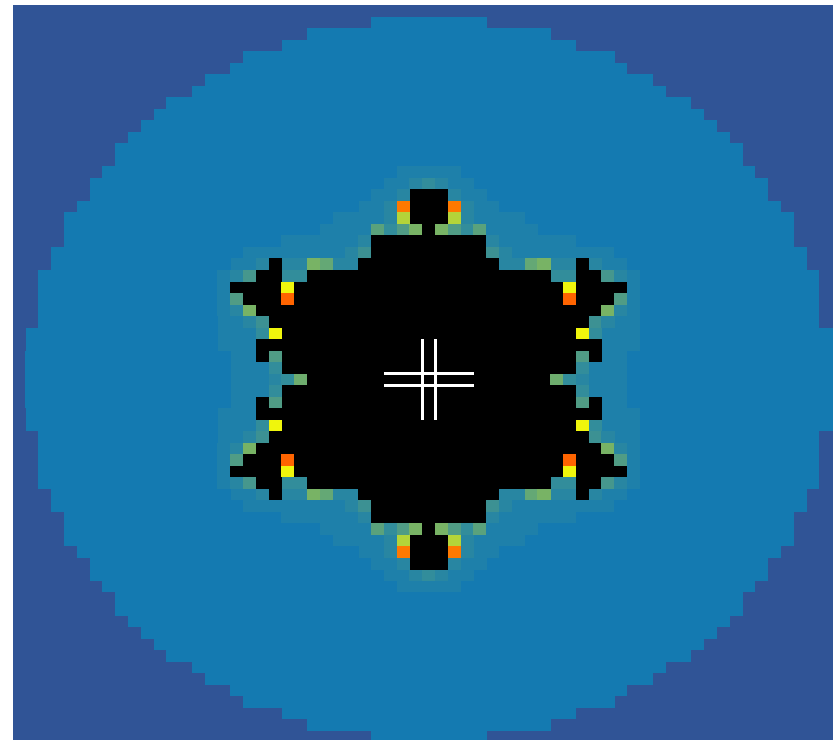
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| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
| Polynomial Order (n) | C | |
| 3 | Real (x) | Imaginary (y) |
| Resolution | 0.0000000000000000 | 0.0000000000000000 |
| 64 | Scale-> | 1.00E+00 |



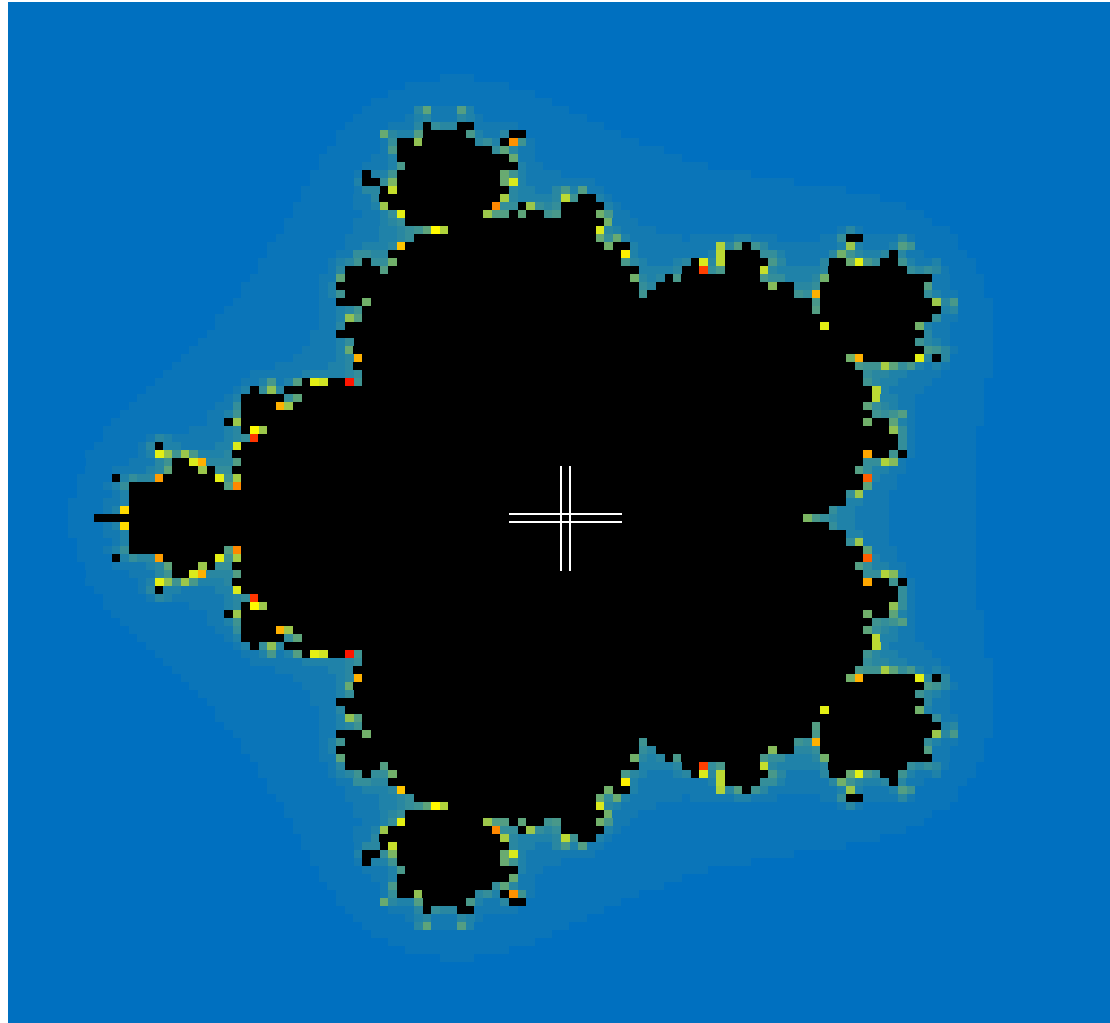
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| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
| Polynomial Order (n) | C | |
| 4 | Real (x) | Imaginary (y) |
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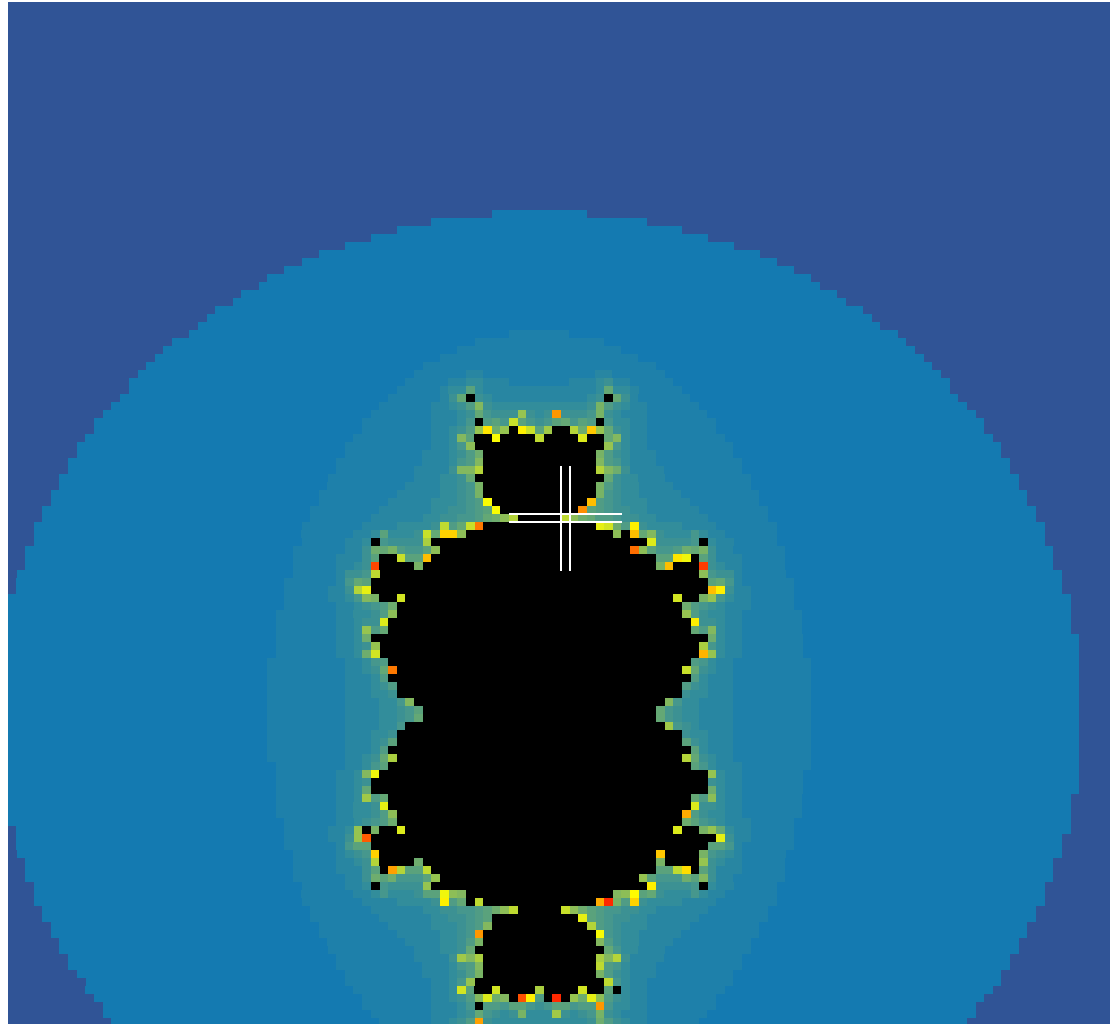
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| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
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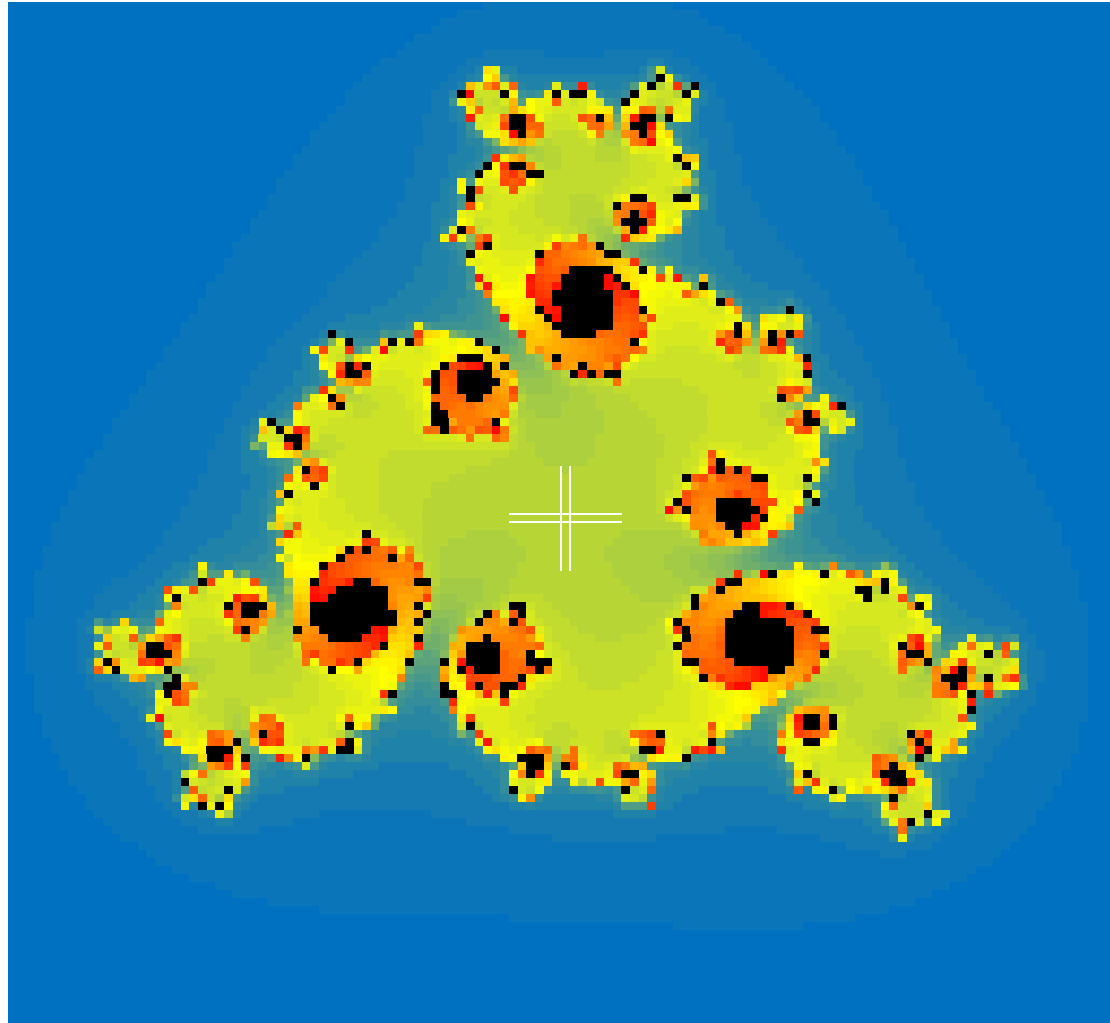
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|---------------------------------|-------------------|-------------------|
| Polynomial Order (n) | C | |
| 6 | Real (x) | Imaginary (y) |
| Resolution | 0.000000000000000 | 0.000000000000000 |
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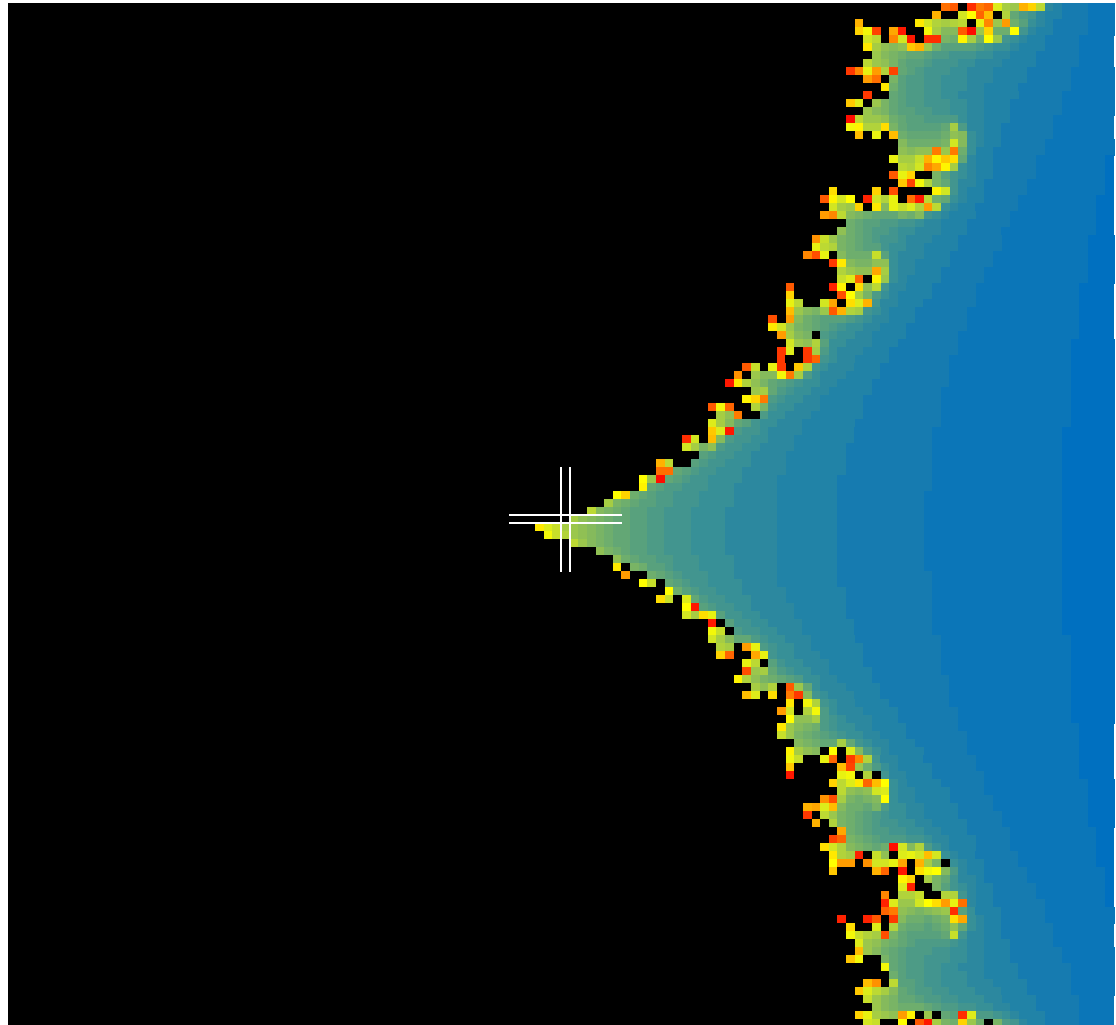
| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
|---------------------------------|-------------------|-------------------|
| Polynomial Order (n) | C | |
| 3 | Real (x) | Imaginary (y) |
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| 128 | Scale-> | 1.00E+00 |



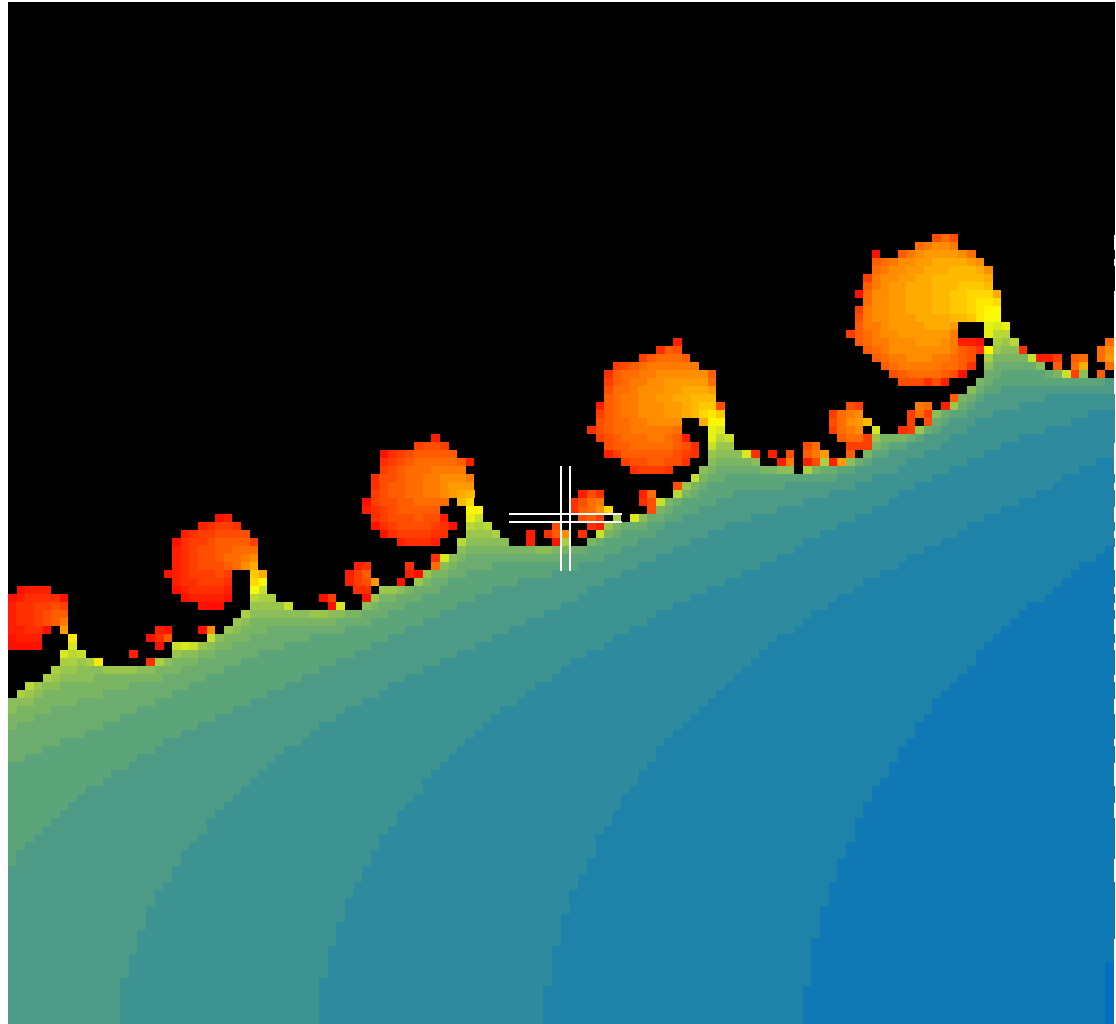
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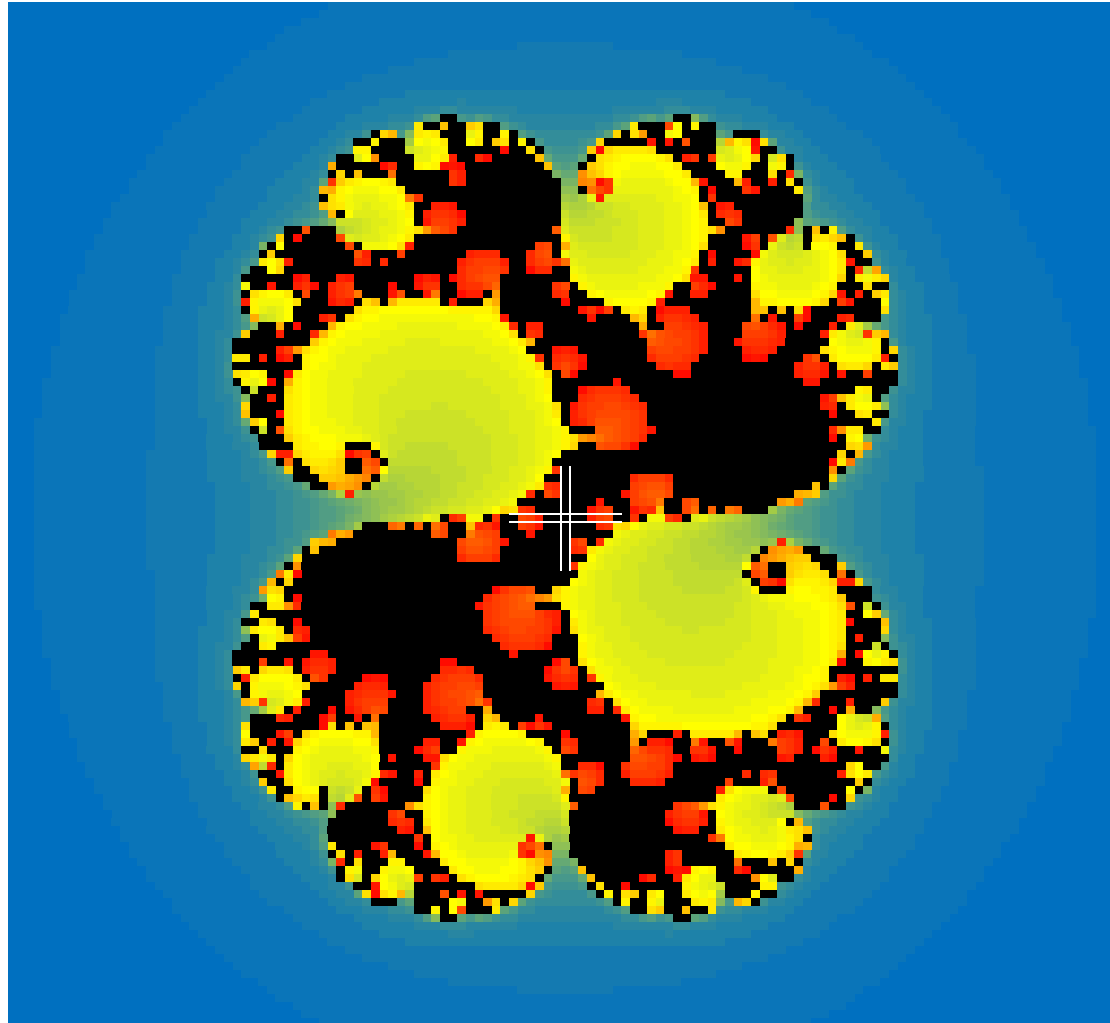
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| Polynomial Order (n) | C | |
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| 128 | Scale-> | 1.00E+01 |



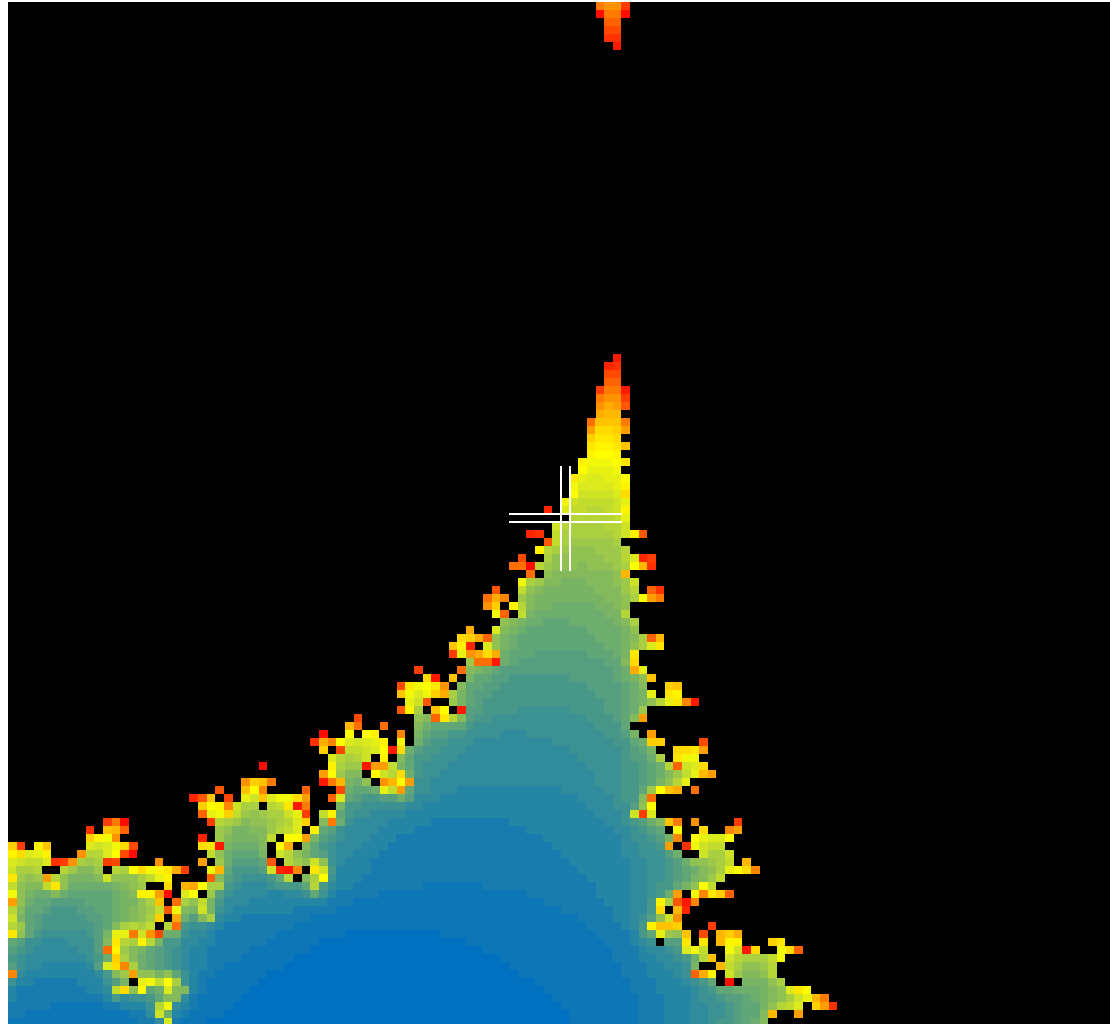
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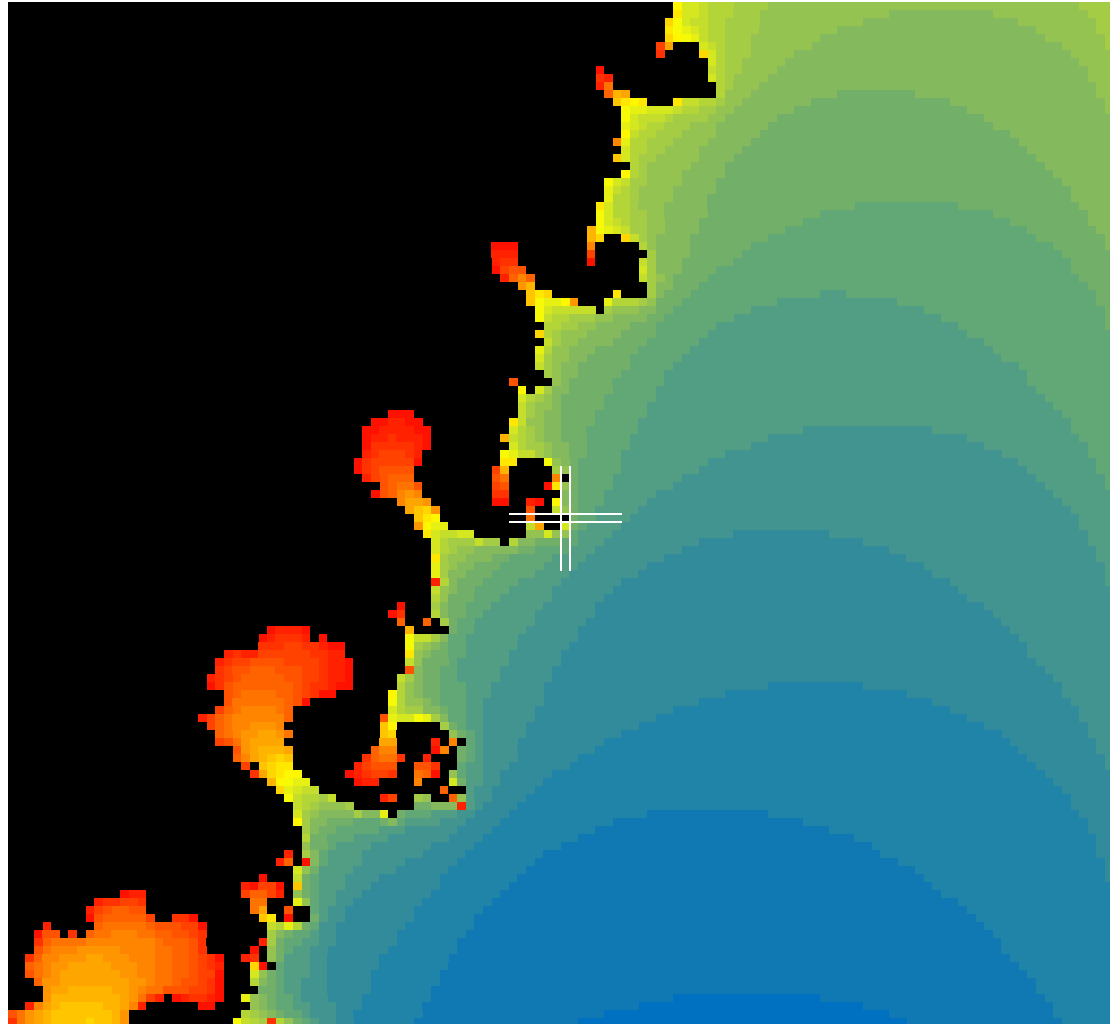
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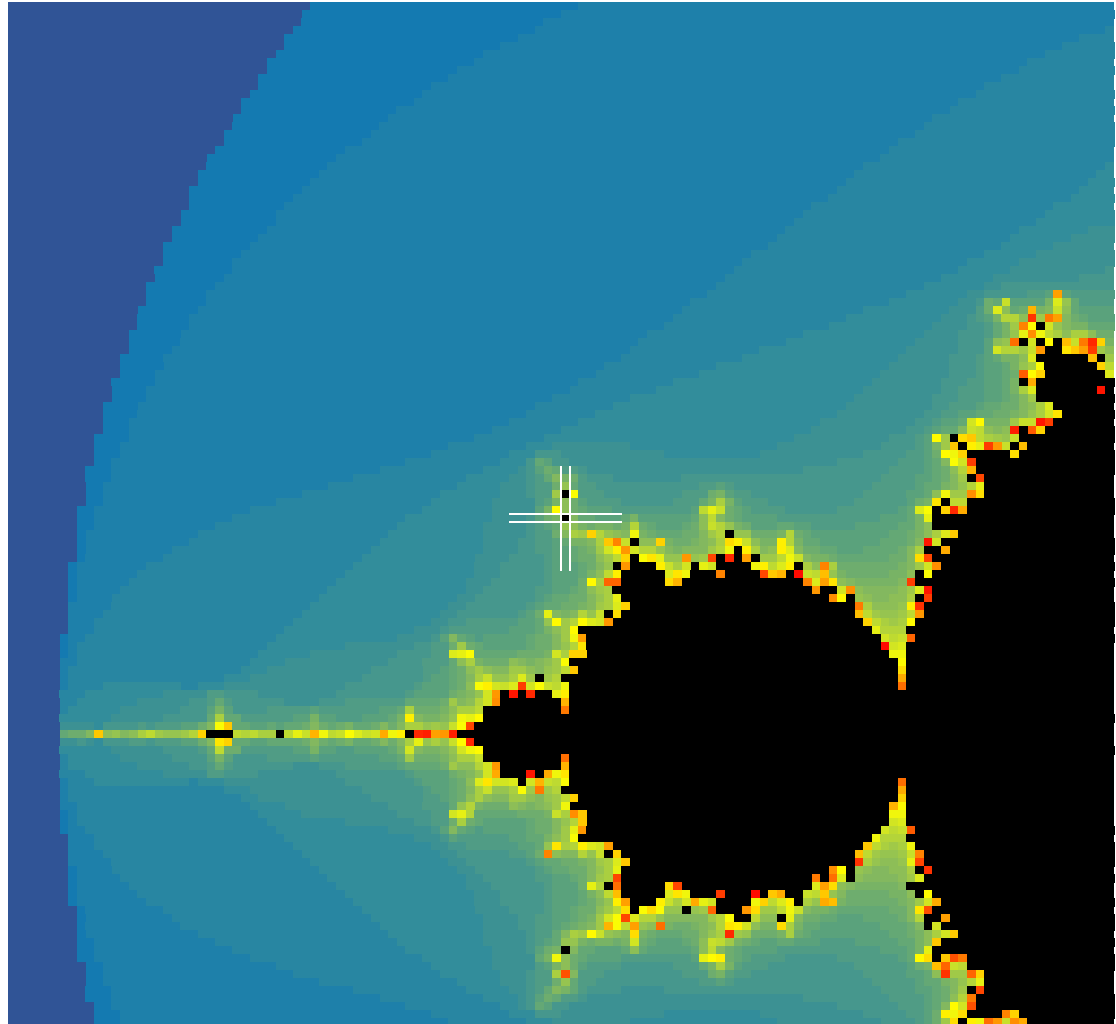
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| 128 | Scale-> | 1.00E+01 |



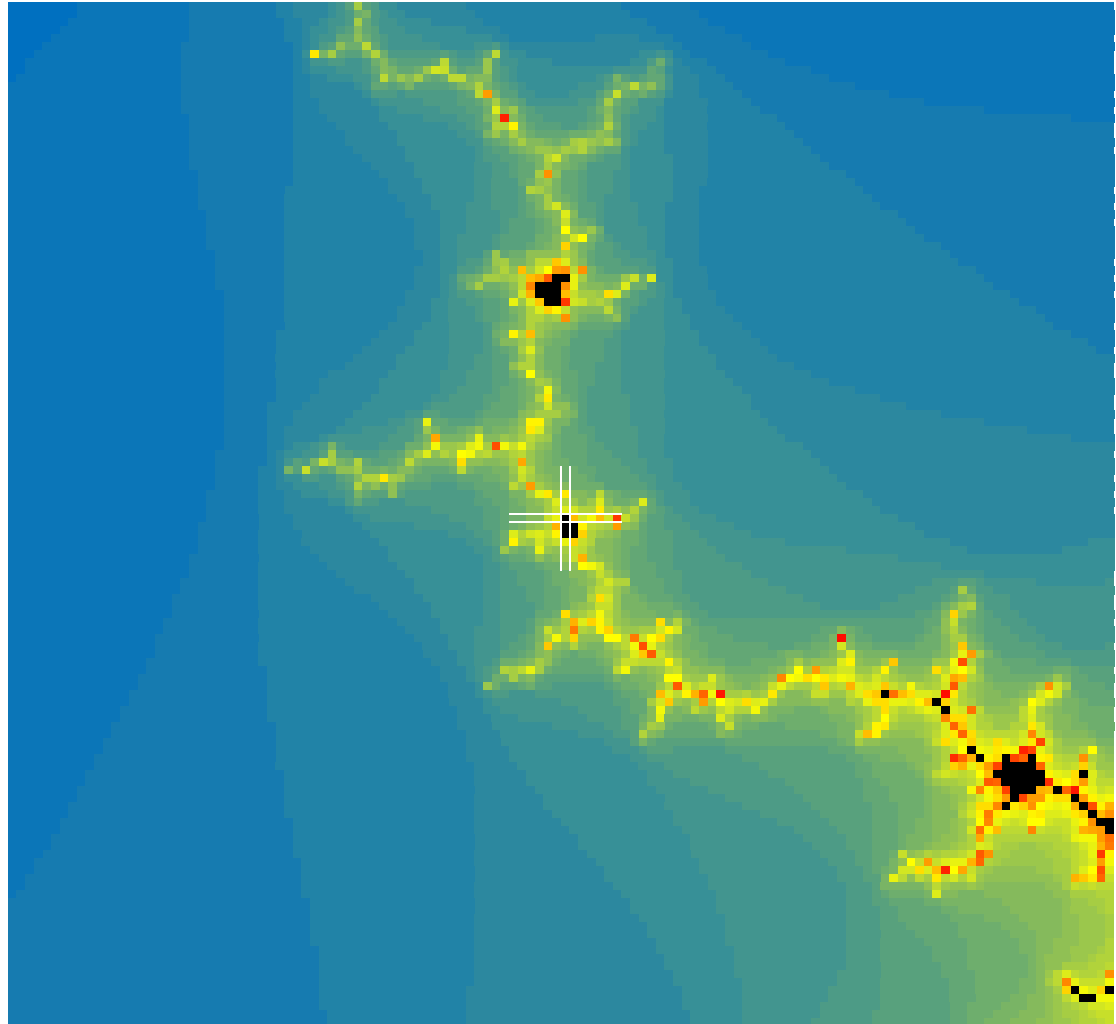
| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
|---------------------------------|-------------------|-------------------|
| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
| Resolution | -0.77092830045081 | -0.12590900476450 |
| 128 | Scale-> | 1.00E+02 |



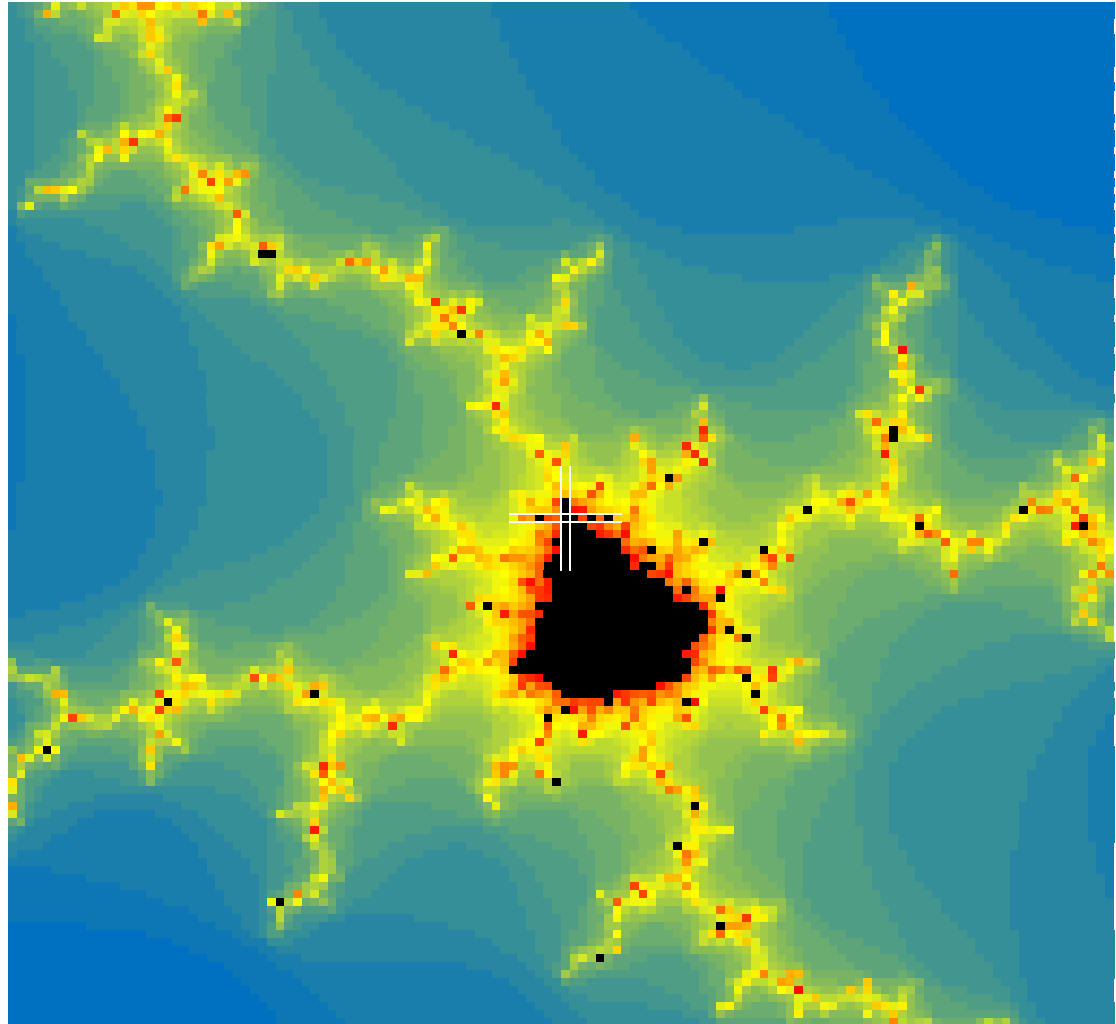
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|---------------------------------|--------------------|-------------------|
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| 128 | Scale-> | 2.50E+00 |



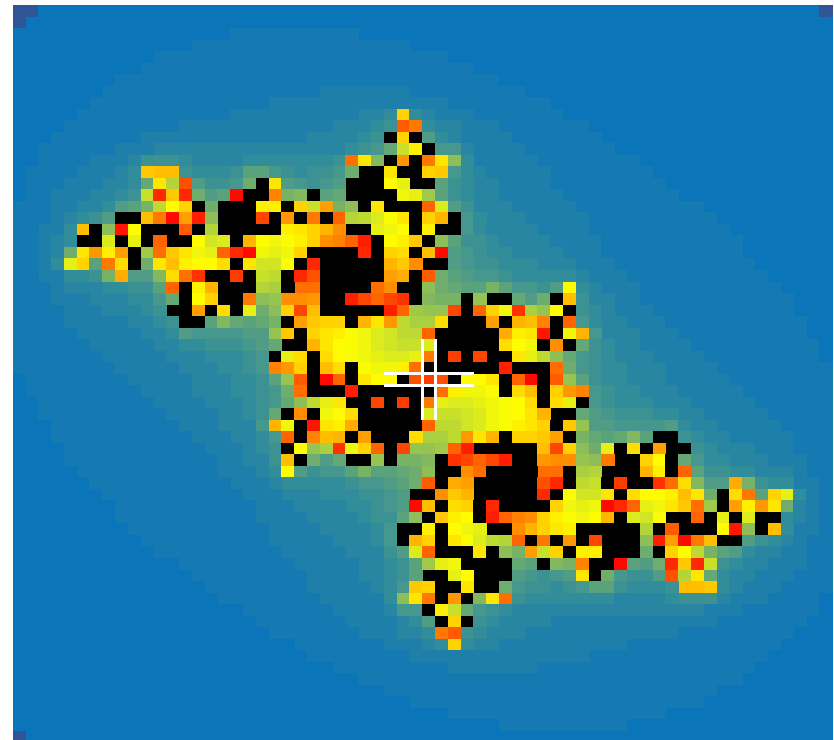
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| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
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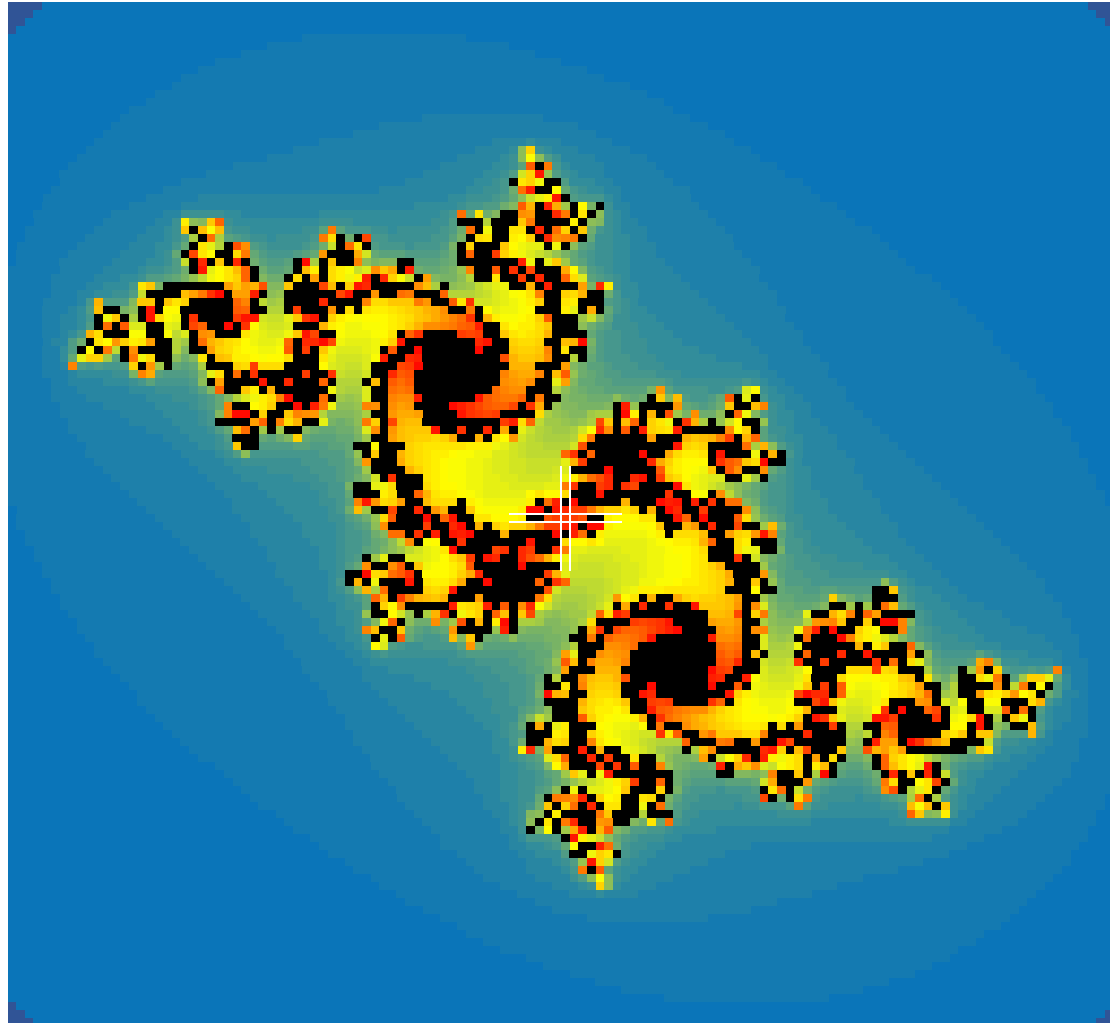
| Mandelbrot Set or Julia Set? -> | | Mandelbrot |
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| 2 | Real (x) | Imaginary (y) |
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| 128 | Scale-> | 2.50E+02 |



| | | |
|---------------------------------|---------------------|--------------------|
| Mandelbrot Set or Julia Set? -> | | Julia |
| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
| Resolution | -0.2040000000000000 | 0.6750000000000000 |
| 64 | Scale-> | 1.40E+00 |



| | | |
|---------------------------------|--------------------|-------------------|
| Mandelbrot Set or Julia Set? -> | | Julia |
| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
| Resolution | -0.204000000000000 | 0.675000000000000 |
| 128 | Scale-> | 1.40E+00 |



| | | |
|---------------------------------|--------------------|-------------------|
| Mandelbrot Set or Julia Set? -> | | Julia |
| Polynomial Order (n) | C | |
| 2 | Real (x) | Imaginary (y) |
| Resolution | -0.204000000000000 | 0.675000000000000 |
| 256 | Scale-> | 1.40E+00 |

